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ABSTRACT

The factor-analytic history of the isolation and identification of the cognitive ability construct known as Number or Numerical Facility is described. Empirical documentation from the psychometric literature is discussed, and examples of items from a variety of tests which have defined and been associated with the Number factor are included to illustrate the type of tasks which require this ability. (Author/BW)

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The Definition and Measurement of Numerical Ability

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THE DEFINITION AND MEASUREMENT OF NUMERICAL ABILITY

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United States Civil Service Commission
Washington, D. C. 20415
August 1977

THE DEFINITION AND MEASUREMENT OF
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ABSTRACT

This report traces the factor-analytic history of the isolation and identification of the cognitive ability construct known as Number or Numerical Facility. Empirical documentation from the psychometric literature is presented and discussed and examples of items from a variety of tests which have defined and been associated with the Number factor are included to illustrate the type of tasks which require this ability.

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The Definition and Measurement of Numerical Ability

One of the first and most definite of the cognitive ability factors to be identified is the Number or Number Facility factor. The clarity of this factor is probably due to the fact that the common characteristic among all tests associated with it is an involvement with numbers (French, 1951). French states that the Number factor represents facility in manipulating numbers in any form; it is not limited to tests involving the four arithmetic operations (p. 225). More recently French, Ekstrom and Price (1963) defined the Number Facility factor as "the ability to manipulate numbers in arithmetical operations rapidly" (p. 28). Tests of the four arithmetic operations (addition, subtraction, multiplication and division) are outstandingly pure measures of the factor (i.e., they have very few and very small loadings on any other factors). Other tests of a numerical nature involving memory for numbers, counting, plotting on numbered coordinates, arithmetic word problems, etc. also have relatively high loadings on Number Facility but they share their variance with factors related to other aspects of these tasks. The Number Facility factor is closely aligned with a second (or higher) order speed factor.

The most recent definition of Number Facility is to be found in Ekstrom, French and Harman (1976). These authors state that Number Facility is "the ability to perform basic arithmetic operations with speed and accuracy. This factor is not (underlining is theirs) a major component in mathematical reasoning or higher mathematical skills" (p. 115). Ekstrom, French and Harman concede that the factor may be broader than simple arithmetic manipulation. There are many factor-analytic studies which support this broader concept, a number of which will be reviewed later in this report. Over 80 studies have reported the existence of some sort of Number factor.

In Guilford's structure of intellect model Number Facility (or N) appears in the Memory for Symbolic Implications (MSI) and Convergent Production of Symbolic Implications (NSI) factors. Different cognitive strategies for retrieval of required information, making proper associations and performing necessary operations may account for "confusion about the nature of numerical facility" (Ekstrom, French and Harman, 1976, p. 115).

Two rather inconclusive, exploratory factor studies on differential mental traits were reported by Garrett (1938) and Woodrow (1939). Primarily Verbal and primarily Numerical components were extracted in each analysis, Garrett having analyzed 9 tests, Woodrow 52. Factor loadings on the Numerical component varied from .52 to .75 for numerical items involving equation relations, number series, multiplication, arithmetic reasoning and vertical addition.

Thurstone and the Primary Mental Abilities

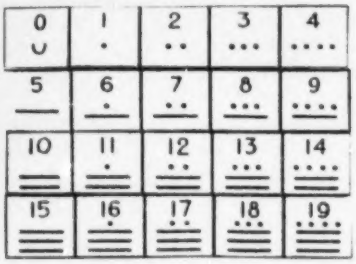
Although a Number factor had appeared previously in the factor-analytic literature, both as a separate and as a group factor, Thurstone's (1938a) investigation of Primary Mental Abilities established "N" as a definite, independent and measurable aspect of human cognitive functioning. From a battery of 56 tests representing a great variety of cognitive tasks, there were eight of a numerical character which had factor loadings greater than .35 on a factor identified as N (for Numerical) by Thurstone. These tests and factor loadings were as follows:

Number Code	.62
Addition	.76
Subtraction	.67
Multiplication	.81
Division	.62
Tabular Completion	.39
Numerical Judgment	.43
Arithmetic Reasoning	.38

The following are examples of items from these tests.

Number Code

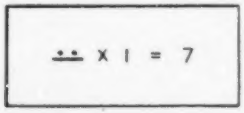
In this test you will be asked to use a number code based on twenty symbols instead of the ten digits to which we are accustomed. There is a symbol for each of the numbers from 0 to 19, as shown below. Notice that a bar means 5 and that a dot means 1. For example, the number 9 is represented by a symbol consisting of a bar and four dots. Zero is represented by a U-shaped symbol as shown.



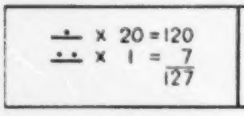
For numbers larger than nineteen, the symbols are combined, one above the other. This is shown in Example 2 below. When there are two symbols, one above the other, the upper one is to be multiplied by 20 and the bottom symbol is to be multiplied by 1. The answer is the sum. Study Example 2.

For numbers larger than 399, three symbols are used, one above the other. The uppermost symbol is multiplied by 400, the next by 20, and the bottom symbol by 1. The answer is the sum. Study Example 3.

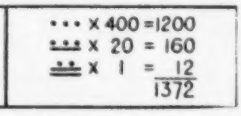
EXAMPLE 1



EXAMPLE 2



EXAMPLE 3



(Thurstone, 1938a, p. 43)

Addition 18 (Add 7 2-digit numbers)
47
84
61
34
78
53

Subtraction 6973 (Subtract 2 4-digit numbers)
4238

Multiplication 697423 (Multiply a 6-digit number by a 1-digit number)
6

Division 7/1861251 (Divide a 7-digit number by a 1-digit number)

(Addition, subtraction, multiplication and division items from Supplement to Thurstone, 1938a, no page number).

Tabular Completion

In the following table there are blank spaces. Fill in the correct numbers in these spaces. Get the necessary information from the rest of the table. Use the space below each table and on the opposite page for figuring.

Year	Marriages		Divorces		
	Number	Increase over Preceding Year	Number	Increase over Preceding Year	Number Granted to Husband
1894	566,161	12,512	37,568	100	12,551
1895	598,855	32,694	40,387	2,819	
1896	613,873	15,018	42,937		14,44
1897	622,350			1,762	14,7
Total	2,401,239	No total	165,591	No total	5

(Thurstone, 1938a, p. 44).

Numerical Judgment

In this test you are shown some arithmetical problems that have already been worked out. Four answers are given for each problem. One of these is always the right answer. You are asked merely to check the right answer. You may use the space on the page for figuring but do not waste time working out the exact answer.

In the first problem below you can readily see that the first number is nearly 4 and the second number is nearly 7. Since 4x7=28, look for the answer that is nearest 28. This is the third answer and it is checked.

4.12395x6.82187 = 7.563327_____
14.012468_____
28.133051__✔____
56.103378_____

(Thurstone, 1938a, p. 46)

Arithmetic Reasoning

At 8 A.M. the barometric pressure was 30.6 and at 9 A.M. the pressure was 31.2. Assuming a constant rate of increase in barometric pressure, at what time was the pressure 31.0? Answer_____A.M.

(from Supplement to Thurstone, 1938a, no page number)

The Addition, Subtraction, Multiplication and Division tests all demand facility in numerical calculation. The problems themselves are simple; in order to receive a high score the examinee has to work rapidly and accurately. Number Code involves some number facility as well as visualization and possibly some learning. It was introduced to diversify the number tests. Even at this early date Thurstone recognized that N might be more basic and general than just facility with numbers. Although arithmetic operations tests were good examples of the kind of ability represented by N, this ability could be required by any numerical task. The last three tests on the list above had higher loadings on factors other than N: Tabular Completion had a loading of .48 on factor I (possibly Induction); Numerical Judgment and Arithmetic Reasoning had loadings of .53 and .58 on factor R (tentatively a reasoning factor) respectively. These three tests obviously require verbal and logical abilities as well as numerical facility. Their lower factor loadings on N indicate that their variance is divided among several factors. Since these measures are more strongly associated with reasoning factors than with the Numerical factor, they are discussed more completely in a report on reasoning abilities (Northrop, 1977).

A further investigation by Thurstone (1938b) of the Perceptual factor identified factor N again with tests of the four arithmetic operations (factor loadings were all greater than .60). The Number factor appeared as clearly as it had in the Primary Mental Abilities analysis, although the population of examinees and the total test battery employed were not the same. The Arithmetic Reasoning test shared its variance with the Number, Verbal and Reasoning factors as it had previously. The existence of factor N became more and more secure in subsequent studies using Thurstone's Numerical Operations and Arithmetic Reasoning tests (Thurstone, 1940; Thurstone and Thurstone, 1941; Goodman, 1943).

A factorial study of number ability by Coombs (1941) investigated several hypotheses concerning the psychological nature of this ability, e.g., does it transcend numerical operations and therefore represent some more fundamental cognitive process? Furthermore, does number ability represent "the rapidity with which well-established associations may be recalled and manipulated?" (p. 184). In order to gain insight into this question Coombs designed several tests which involved the manipulation of a symbolic system. These tests can best be described with an example from one of them:

ABC Test

Only three letters A, B and C, are used in this test. They are combined in various ways, but there are two rules:

(1) A combination of any two different letters is equal to the third letter, e.g., AB=C, AC=B, CA=B, etc.
(2) If a letter is followed by itself, the combination is equal to that letter, e.g., AA=A, etc.

Example: ABA=B because -
AB=C and then, CA=B
(Coombs, 1941, p.168)

Another test of exactly the same nature as the above utilized three geometric designs rather than letters of the alphabet.

In addition, three measures were incorporated into the test battery for the purpose of determining whether or not numerical tests of a non-manipulative character would also identify the Number factor. One of these consisted of 45 columns, with 29 three-digit numbers in each. The examinee's task was to find and mark each number in a column which was the same as the number at the top of that column. An example follows.

Identical Numbers

634	876	795
693	643	583
850	328	795
634	932	189
513	879	342
398	375	795
696	470	896
634	697	247
574	876	319
628	294	468
634	982	543
.	.	.
.	.	.

(Coombs, 1941, p. 172)

Lastly, the hypothesis that the "operation characterizing the tests defining the number factor is in the nature of a serial response, in that each response to a pair of numbers leads to the next response" (p. 164), was tested by using addition problems of varying length (two-digits, three-digits and four-digits). If the serial response hypothesis was true, the addition of four-digits would be the best measure of the Number factor (i.e., these problems would have higher factor loadings on N than would the two-or three-digit problems).

In all, the battery consisted of 18 experimental tests and 16 reference tests for Thurstone's seven Primary Mental Abilities.

The results of Coombs study were interesting, if not altogether conclusive. The measures with factor loadings greater than .20 on the Number factor were as follows:

Two-digit Addition	.74
Three-digit Addition	.72
Four-digit Addition	.66
Multiplication(Thurstone)	.64
Addition (Thurstone)	.45
Identical Numbers	.22

The hypothesis that the underlying characteristic of number ability is in the form of a serial response was not supported, since the factor loading of four-digit addition was lower, not higher, than the loadings for three-and two-digit addition. Of the measures designed to determine whether numerical tests which did not involve number manipulation would also identify the Number factor, only one, Identical Numbers, had a reportable factor loading (.22) on N. Certainly no definite conclusion can be drawn from this isolated piece of data. Although the tests involving manipulation of symbols (ABC test, Forms test) achieved loadings of less than .10 on the Number factor, they did correlate with the arithmetic operations tests which identified Number ability. The ABC test had higher correlations with the several Addition and Multiplication measures (around .45) than did the Forms test (using geometric shapes), for which the same correlations were around .35. Coombs drew several conclusions from the analysis, the most interesting of them being (p. 188-189):

1) The Number factor is most clearly identified by very simple number tests.
2) A test involving manipulation of a symbolic system is a better measure of number ability the more familiar the symbolism.
3) The results are in agreement with the hypothesis that number ability is characterized by a facility in manipulating a symbolic system according to a specified set of rules.

Relative to the third conclusion above, recall that Thurstone's Number Code test had a factor loading of .62 on factor N, exceeded only by the factor loadings of Addition, Subtraction and Multiplication (Thurstone, 1938a).

A number of additional studies by different authors continued to confirm Thurstone's N as a primary in the human ability structure. Zimmerman (1953) revised the factor rotation solution for Thurstone's original Primary Mental Abilities battery in order to eliminate a large amount of residual variance. Factor loadings for the Numerical factor were essentially unchanged by this technique, which subjected the original seven factors to further rotation. A similar analysis was undertaken by Wrigley, Saunders and Neuhaus (1958), who used a completely different rotational method with Thurstone's correlation matrix. Again, there was

remarkable agreement between factor loadings for the tests which identified N in both studies.

The Numerical factor thus far, as identified primarily by tests of the four arithmetic operations, seems exceedingly stable, i.e., it is invariant over different examinee populations, over test batteries differing both in size and composition, and over various factor-analytic and rotational techniques.

Army Air Force and Related Studies

During World War II the Army Air Force Aviation Psychology Program developed and carried out many factor-analytic studies on numerous mathematics tests which were a part of the air crew classification batteries. "Job-analysis findings indicated that mathematics [was] extremely important to navigation and that measures of achievement or proficiency in mathematics should be good predictors of navigational success" (Guilford and Lacey, 1947, p. 73). Tests of two types--general mathematics and numerical computations--were included in the classification batteries. Examples from those tests most frequently used and factor-analyzed are given below.

Mathematics A (CI702E)

This test was designed to measure competence in mathematics. In general, a student who has completed high-school algebra should be equipped to solve most of the problems.

1) $(3x-1)(\frac{2}{2}x+2) =$
   - A. $6x^2-x+2$
   - B. $6x^2+4x-2$
   - C. $6x^2-4x-1/2$
   - D. $3x^2-x+3$
   - E. x^2+2x-6

$R = c^2d^3$. If $c = 2$ and $d = 3$, then $R =$
   - A. -108
   - B. -72
   - C. 36
   - D. 72
   - E. 108

(Guilford and Lacey, 1947, p. 74).

Numerical Operations (CI701B)

Part I (examinee marked given answer either C (correct) or W (wrong))

11 + 19 + 22 = 52 C= W=

Multiply: 139 × 7 = 973 C= W=

Add: 12 + 19 + 28 = 59 C= W=

Part II (examinee chose correct answer)

Subtract: 63-38: 25 21 29 32 26

Divide: 233 ÷ 7: 39 37 33 2/7 37 2/7 35

Because of the extremely low absolute difficulty of the problems, the time limits were made short, thus making the test highly speeded.

(Guilford and Lacey, 1947, p. 80)

Numerical Approximation (CI706A)

This test is designed to measure the student's ability to estimate quickly the accuracy of results of fairly simple arithmetic operations. It differs from numerical operations in that emphasis is placed upon estimation rather than upon computation. Bombardiers and navigators frequently must make computations under pressure and in limited time. It is thus important that they be able to check their work quickly. Gross errors--the most serious ones--are usually detectable because of the unreasonableness of the results. For example, misplaced decimal points and similar errors should be apparent to one who sees the problem as a whole and is able to estimate within reasonable limits the results of arithmetic operations. The cues to discrepancies are many--numbers of digits, size of first and last digits, position of decimal points, and the like. To minimize the sheer

numerical-operation component, directions encourage the examinee to estimate the answers roughly, not taking time to compute them. The time limit also is set so short that those who stop to compute the exact answers inevitably fail to complete enough items to obtain a good score.

8,000 X (.96288-.94208) = _____
A. 20.
B. 80.
C. 160.
D. 208.
E. 344.

(Guilford and Lacey, 1947, p. 84-85).

Mathematics B

This test was an unweighted combination of an Arithmetic Reasoning test[1] and the Numerical Approximation test above. The following is an example of an item from Arithmetic Reasoning (CI206C):

If a plane is to fly 132 miles in 45 minutes, what must be its average speed?
A. 146.7 m.p.h.
B. 164 m.p.h.
C. 165 m.p.h.
D. 176 m.p.h.
E. 182 m.p.h.

(Guilford and Lacey, 1947, p. 90)

Table 1 presents a summary of the factor loadings of the mathematics tests from seven Air Force studies. The table was compiled from French's (1951) survey of 15 Air Force factor analyses reported in their entirety in Guilford and Lacey (1947). An examination of Table 1 reveals that the Numerical Operations tests consistently achieve the highest loadings on the Number factor and do not have loadings on either the Verbal or Reasoning factors. This is in agreement with Thurstone's findings. The other mathematics tests share their variance with Reasoning and Verbal ability, also consistent with previous observations. In six of eight additional Air Force factor-analytic studies, wherein the test batteries did not include a Numerical Operations test, a Numerical factor was not isolated even though one or more of the other mathematics tests was present. The failure of N to appear at all was no doubt due to an insufficient number of tests of a numerical nature in the battery. The Number factor was present in the remaining two studies, identified by Arithmetic Reasoning in combination with one or more tests which assessed the ability to read dials, tables and/or scales, or to plot numbers on a set of coordinates.

The Air Force Numerical Operations and Arithmetic Reasoning tests continued to be used by other investigators in further explorations of many areas of cognitive ability (e.g., perception, memory, reasoning). Loadings for these tests on the Numerical factor continued to fall in the same range as previously reported--.60 to .80 for Numerical Operations; .30 to .50 for Arithmetic Reasoning (Dudek, 1948; Comrey, 1949; Michael, 1949; Adkins and Lyerly, 1952; Guilford, Fruchter and Zimmerman, 1952; Roff, 1952; Kelley, 1964; Very, 1967).

The stability of the Numerical factor as referenced by these Air Force tests was demonstrated by Zachert and Friedman (1953) who found that factor loadings on N for Numerical Operations varied from .69 to .78 in four separate analyses of the same Aircrew Classification battery; factor loadings for Arithmetic Reasoning were even less deviant, ranging from .35 to .38.

Numerical Operations tests were frequently diversified to include other aspects of computation such as decimals, fractions and more than one of the four arithmetic operations in a single problem. The United States Employment Service (French, 1951) factor-analyzed a battery of occupational aptitude tests which included measures of the examinee's ability to add, subtract, multiply and divide decimals and fractions. These two tests identified a Numerical factor, with loadings of .51 and .39, along with an Arithmetic test (the four operations used on whole numbers), Reasoning problems (verbally expressed) and Number Comparisons (judge the sameness of numbers

[1]Arithmetic Reasoning tests have higher factor loadings on Reasoning and are used as reference tests for the General Reasoning factor. They are included and discussed briefly in a report on the Definition and Measurement of Reasoning (Northrop, 1977). Since, however, these tests do have appreciable loadings on the Number factor, they appear in this paper as well by virtue of their contribution to the assessment of numerical ability.

TABLE 1

Summary of Average Factor Loadings for Air Force Mathematics Tests

Test	Factor		
	Number	Verbal	Reasoning
Numerical Operations Part I (7 studies)	73 (63-81)*		
Numerical Operations Part II (6 studies)	75 (63-81)		
Mathematics A (2 studies)	46 (42-51)	45 (37-53)	25
Arithmetic Reasoning (6 studies)	48 (38-60)	34 (27-40)	52 (47-63)
Numerical Approximation (1 study)	51	34	55
Mathematics B (2 studies)	62 (59-65)	28 (27-28)	43 (36-50)

*range of the factor loadings

in pairs). Factor loadings for these last three tests were .59, .45 and .41 respectively. Fruchter (1952) analyzed an extensive test battery of aptitude and achievement tests from numerous sources. In addition to the Air Force Numerical Operations and Arithmetic Reasoning tests, the selection of variables included the Army General Classification Arithmetic Computation test (addition, subtraction, multiplication, division, percentages, fractions, decimals and proportions) and Arithmetic Reasoning test (verbally and diagrammatically presented problems). All had factor loadings of .58 to .68 on the Numerical factor. In this analysis a socioeconomic and cultural questionnaire and a measure of academic information (current events, etc.) had no association with the numerical measures.

The hypothesis advanced by Coombs (1941) that numbers do not necessarily define the boundaries of a numerical task (measures involving symbols such as letters or geometric shapes may also represent numerical facility) was explored further by Davis (1956) and DeMille (1962). In these two investigations the Wechsler-Bellevue Digit Symbol test was found to have about the same factor loading on Number Facility (.63) as the Air Force Numerical Operations test (.61). The Digit Symbol test was first used in the old Army Beta examination (Yoakum and Yerkes, 1920). A meaningless symbol was associated with each of the numbers from 1 through 9 as follows:

1	2	3	4	5	6	7	8	9
−	⋂	⊐	L	∪	o	^	X	=

Below these given symbols were rows of boxes containing, randomly, the numbers 1-9. The examinee's task was to place the correct symbol below each number as rapidly as possible. The following is one row from the test:

6	3	1	5	4	2	7	6	3	8	7	2	9	5	4

(Yoakum and Yerkes, 1920, p. 280)

The Research of Guilford and Associates

Factor-analytic studies carried out by J. P. Guilford and his many associates in the cognitive areas of spatial relations and visualization (Michael, Guilford and Zimmerman, 1951), reasoning (Green, Guilford, Christensen and Comrey, 1953; Guilford, Christensen, Kettner, Green and Hertzka, 1954; Kettner, Guilford and Christensen, 1956), creative thinking (Wilson, Guilford, Christensen and Lewis, 1954) and planning abilities (Berger, Guilford and Christensen, 1957) repeatedly isolated the already firmly established Numerical factor. Numerical Operations and Arithmetic Reasoning tests continued to present factor loadings of a magnitude consistent with previous findings. The latter test also continued to appear in the General Reasoning factor as well as in the Numerical. Frequently loadings on the two factors were about equal (.40 to .50) indicating that the ability to manipulate numbers is as important in solving the problems presented in this type of test as is the ability to think logically. It is true of arithmetic reasoning items that if the logic required for the solution is complex and the numerical calculations simple, the test will have a considerably higher loading on the General Reasoning than on the Numerical factor. If the reverse is true, reasoning ability may not be required at all and the test will merely be one of numerical facility.

As Guilford's structure of intellect model of cognitive abilities developed, the primary mental abilities were fragmented into a host of narrower and much more specific structural units. Reasoning processes fell into the realm of the intellective activity referred to as cognition--the discovery, awareness or recognition of information in its various forms (figural, symbolic, semantic). Arithmetic Reasoning went, factorially, to Cognition of Semantic Systems (CMS). Guilford and Hoepfner (1971) cite at least four studies in which the Air Force Arithmetic Reasoning test had factor loadings from .33 to .57 on this factor. The Numerical Facility factor was identified with the SI factors Convergent Production (achievement of the best conclusion from given information) of Symbolic Implications (NSI) and to Memory for Symbolic Implications (MSI). Guilford has not made extensive use of classic Numerical Operations tests in his search for reference tests for the structure of intellect factors. However, in two studies (Guilford and Hoepfner, 1971) factor loadings on NSI of .32 & .36 were reported; in two different analyses, loadings of .35 and .50 on MSI were reported. The whereabouts of the remainder of the variance in Numerical Operations tests is unclear.

There are some psychologists who think that the structure of intellect model categorizes cognitive ability into units which are too narrow to be assessed from a practical standpoint, i.e., when one is trying to measure the aptitudes necessary for successful performance in various occupations. Hakstian and Cattell (1974) measuring abilities on a broader basis, and using all new tests, found many of Thurstone's primaries (V, N, S, I and W) to be essentially unchanged since their initial identifications. Employing a battery of 57 tests suggested by French, Ekstrom and Price (1963) and Guilford and Hoepfner (1971), these authors extracted 19 ability factors, among them the Numerical or Number factor, which was defined as it repeatedly has been in the past, i.e., facility in manipulating numbers. "This function is factorially distinct from both (1) arithmetic reasoning (tests of which tend to load on N as well as on both Induction and a Deductive Reasoning factor when it is found" (Hakstian and Cattell, 1974, p. 146). In other words, numerical facility and reasoning are two separate and distinct aspects of quantitative tasks.

Summary and Conclusions

The Numerical or Number factor has been well documented in many factor-analytic studies using a diversity of examinee populations and test batteries (in terms of both number and kinds of variables). There has been remarkably consistent agreement among numerous investigations as to the nature of the factor - it is concerned with the proficient manipulation of numbers. The purest measures of the factor are speeded tests requiring simple but rapid calculations using the four basic arithmetic operations (addition, subtraction, multiplication and division). More complex tests such as arithmetic problems stated verbally (arithmetic reasoning tests) have moderate factor loadings on N as well as on reasoning factors (primarily General Reasoning). The factor loadings of these quantitative tests on the Number factor are directly proportional to the amount of numerical manipulation involved, and conversely, the more difficult the reasoning aspect of the problems, the higher the factor loadings on General Reasoning.

To assess quantitative ability in just the specific sense of speed with which numbers are manipulated (that is, with very simple computation tests) does not get at the broader aspect of the concept, which embraces proficiency in <u>both</u> numerical manipulation and logical thinking. Broad constructs must be measured with variables of breadth (or factorial complexity). For this reason most standardized measures for the assessment of quantitative aptitude consist of computational exercises as well as mathematical problems involving reasoning.

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